Lab 1 : Entangled Photon and Bell's Inequalites

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In this experiment we create entangled photons by using two BBO crystals and verify various properties of entangled photons. We observe the cosine squared dependence of coincidence on angles of polarization with visibility close to 100% as expected. We test the Bell's Inequalities and get $S = 2.64 \pm 0.08$ which violates the inequalities as predicted.

1. Background and Theoretical explanation

1.1) Historical Review of Entanglement

In development of Quantum Theory in the early of 20th century, one successful step in physics is discovering Schrödinger equation which can precisely predict the experimental results like no one has ever done before. But one consequence from the equation is pointed out by A. Einstein and his colleagues in their infamous paper [1], which is well-known as EPR paradox, that this result in turn seems to deny locality[1] and inconsistent which his famous theory, General Relativity. This consequence leads to the instantaneous communication between two parts of the system regardless of space-time dependence. In his famous paper, Einstein suggested that quantum theory has not yet completed, there are still some missing piece of jigsaw in the theory and also proposed a theory, so called (global) hidden variable, to fill out the hole in the theory. On the other side, Bohr, who is one of the pioneers in formulating Quantum Theory, suggested that this kind of system cannot be considered as a two system separately since the first place. Consequently, no matter how far they are apart, both particles don't need any kinds of communication and the Quantum Theory is complete. At that time no one could verify Einstein's proposition or Bohr's suggestion until 1964, J.S. Bell published his famous mathematical inequalities which provide us the upper bounded of measurement of any abstract system which behaves locally [2]. This inequalities, in 1969, is re-derived by J. Clauser, M. Horne, A. Shimony, and R. Holt for an optical system [3]. Specifically they derived a new version of Bell's inequalities for a measurement of coincidence of two different polarization photons. About a decade later, A. Aspect succeeded in carrying out the experiment to verify this inequalities and he found that the inequalities is violated in an experiment with entangled system which implies that entanglement does not behave locally and Einstein's idea about hidden variable is not correct[4]. Entangled state is a state of a system of particles which cannot be described by the state of each particle individually. In mathematical language, the state space of the system cannot be factorized into the state space of each particle.

$$|\Psi_{12}\rangle \neq |\Psi_1\rangle \otimes |\Psi_2\rangle \tag{1}$$

[1] Locality in this context refers to there are nothing can travel with speed greater than light and two parts of the system separated in space can communicate to each other with at most at light speed. Having this constraint violated is referred as non-locality.

1.2)Theoretical Explanation

For simplicity, let consider the entanglement of two particles of two-state system and for convenience in the future referring, let the system be the two-photon entanglement in polarization. This system each photon is characterized by its polarization, denote vertically polarized by $|V\rangle$ and horizontally polarized by $|H\rangle$ and use the convention that the bra or ket on the left is the state for photon number one and the bra or ket on the right for the other photon. In this case we can choose

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|V\rangle|V\rangle + |H\rangle|H\rangle)$$

(2)

Notice that this state is not equal to the state obtained by performing the outer product of two state spaces of each particle

$$|\acute{\Psi}\rangle = \frac{1}{2}(|V\rangle|V\rangle + |V\rangle|H\rangle + |H\rangle|V\rangle + |H\rangle|H\rangle)$$

(3)

In CHSH inequalities, the system is considered about measurement of coincidence count from two detectors. The inequalities states that

$$S = \left|E(a,b) - E\left(a,b^{'}\right)\right| + \left|E\left(a^{'},b\right) + E\left(a^{'},b^{'}\right)\right|$$

(4)

where

$$E(\alpha,\beta) = \frac{N(\alpha,\beta) + N(\alpha_{\perp},\beta_{\perp}) - N(\alpha_{\perp},\beta) - N(\alpha,\beta_{\perp})}{N(\alpha,\beta) + N(\alpha_{\perp},\beta_{\perp}) + N(\alpha_{\perp},\beta) + N(\alpha,\beta_{\perp})}$$

(5)

α,β are angles of two polarizers which are placed in front of detectors and the subscribe means the angle which is perpendicular to that angle

$N(\alpha,\beta)$ is a coincidence rate when two polarization is set at angle α,β

Speak roughly, the quantity $E(\alpha,\beta)$ relates to a probability for measuring two photons at several given polarization. This form, CSHS inequalities, of the Bell's inequalities will give the upper bounded of the measurement of classical system as 2. A violation of this inequality implies the system does not behave locally. By verifying this violation we can show that entanglement is a system with behave non-locally as suggested by quantum theory and local hidden variable theory is not true. Before we can simplify this expression, we first show one useful property of the entangled state that is the state is independent of the choice of basis using in writing down the state. To show this we first note that changing the basis corresponds to rotating the state, hence

$$\begin{pmatrix} |V\rangle \\ |H\rangle \end{pmatrix} = \begin{pmatrix} Cos\theta & -Sin\theta \\ Sin\theta & Cos\theta \end{pmatrix} \begin{pmatrix} |\acute{V}\rangle \\ |\acute{H}\rangle \end{pmatrix}$$

$$\begin{pmatrix} |V\rangle \\ |H\rangle \end{pmatrix} = \begin{pmatrix} Cos\theta|\acute{V}\rangle - Sin\theta|\acute{H}\rangle \\ Sin\theta|\acute{V}\rangle + Cos\theta|\acute{H}\rangle \end{pmatrix}$$

(6)

Therefore (2) becomes

$$\begin{aligned} |\Psi\rangle &= \frac{1}{\sqrt{2}}\Big(\big(Cos\theta|\acute{V}\rangle - Sin\theta|\acute{H}\rangle\big)\big(Cos\theta|\acute{V}\rangle - Sin\theta|\acute{H}\rangle\big) + (Sin\theta|\acute{V}\rangle + Cos\theta|\acute{H}\rangle)(Sin\theta|\acute{V}\rangle + Cos\theta|\acute{H}\rangle)\Big) \\ &= \frac{1}{\sqrt{2}}(Cos^2\theta|\acute{V}\rangle|\acute{V}\rangle - Sin\theta Cos\theta|\acute{H}\rangle|\acute{V}\rangle - Sin\theta Cos\theta|\acute{V}\rangle|\acute{H}\rangle + Sin^2\theta|\acute{H}\rangle|\acute{H}\rangle + Cos^2\theta|\acute{H}\rangle|\acute{H}\rangle \\ &\qquad + Sin\theta Cos\theta|\acute{H}\rangle|\acute{V}\rangle + Sin\theta Cos\theta|\acute{V}\rangle|\acute{H}\rangle + Sin^2\theta|\acute{V}\rangle|\acute{V}\rangle) \\ &= \frac{1}{\sqrt{2}}((Cos^2\theta + Sin^2\theta)|\acute{V}\rangle|\acute{V}\rangle + (Cos^2\theta + Sin^2\theta)|\acute{H}\rangle|\acute{H}\rangle) \\ &= \frac{1}{\sqrt{2}}(|\acute{V}\rangle|\acute{V}\rangle + |\acute{H}\rangle|\acute{H}\rangle) \end{aligned}$$

(7)

This result allows us to use any two angles which are perpendicular to each other as a basis to represent the entanglement state. Namely, in the expression for $E(\alpha,\beta)$ which consists of two sets of two angles which are perpendicular to each other. We can call it vertical and horizontal. And any results we get on basis $|H\rangle$,$|V\rangle$ are true in general.

With this fact, $E(\alpha,\beta)$ can be written as

$$E(\alpha,\beta) = P_{VV}(\alpha,\beta) + P_{HH}(\alpha,\beta) - P_{VH}(\alpha,\beta) - P_{HV}(\alpha,\beta)$$

(8)

where

P_{VV} is the probability for measuring both photons as vertically polarized photons

P_{HH} is the probability for measuring both photons as horizontally polarized photons

P_{VH}, P_{HV} are the probabilities for measuring one photon as vertically polarized photon and the other as horizontally polarized photon. The order of subscribes represent the result from each detector. They can be explicitly written as following

$$P_{VV} = \frac{N(\alpha,\beta)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp)}$$

$$P_{HH} = \frac{N(\alpha_\perp,\beta_\perp)}{N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp) + N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp)}$$

$$P_{HV} = \frac{N(\alpha_{\perp}, \beta)}{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) + N(\alpha_{\perp}, \beta) + N(\alpha, \beta_{\perp})}$$

$$P_{VH} = \frac{N(\alpha, \beta_{\perp})}{N(\alpha, \beta) + N(\alpha_{\perp}, \beta_{\perp}) + N(\alpha_{\perp}, \beta) + N(\alpha, \beta_{\perp})}$$

(9)

In measurement, (9) can be written as

$$P_{VV} = |\langle \mathrm{V}|\langle \mathrm{V}|\Pi(a,b)\ |BBO\rangle|^2$$

$$P_{HH} = |\langle \mathrm{H}|\langle \mathrm{H}|\Pi(a,b)\ |BBO\rangle|^2$$

$$P_{HV} = |\langle \mathrm{H}|\langle \mathrm{V}|\Pi(a,b)\ |BBO\rangle|^2$$

$$P_{VH} = |\langle \mathrm{V}|\langle \mathrm{V}|\Pi(a,b)\ |BBO\rangle|^2$$

(10)

Where $\Pi(a,b)$ is an operator which represents the two polarizers in front of the detectors that rotate the state of photons by angle a, b denote the angle of each polarizer which respects to a vertical direction, a for the first detector and b for the second.

$$\begin{pmatrix} |V\rangle|V\rangle \\ |H\rangle|V\rangle \\ |V\rangle|H\rangle \\ |H\rangle|H\rangle \end{pmatrix}$$

(11)

$\Pi(a,b)$ takes the form

$$\begin{pmatrix} CosaCosb & -SinaCosb & -CosaSinb & SinaSinb \\ SinaCosb & CosaCosb & -SinaSinb & -CosaSinb \\ CosaSinb & -SinaSinb & CosaCosb & -SinaCosb \\ SinaSinb & CosaSinb & SinaCosb & CosaCosb \end{pmatrix}$$

(12)

Therefore for entangled state in (2) takes the form

$$|BBO\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

(13)

(12) and (13) give

$$\Pi(a,b)\,|BBO\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} CosaCosb + SinaSinb \\ SinaCosb - CosaSinb \\ CosaSinb - SinaCosb \\ SinaSinb + CosaCosb \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} Cos(a-b) \\ Sin(a-b) \\ -Sin(a-b) \\ Cos(a-b) \end{pmatrix}$$

(14)

Therefore (10) gives

$$P_{VV} = |\langle V|\langle V|\Pi(a,b)\,|BBO\rangle|^2 = \left\{(1 \quad 0 \quad 0 \quad 0)\frac{1}{\sqrt{2}}\begin{pmatrix} Cos(a-b) \\ Sin(a-b) \\ -Sin(a-b) \\ Cos(a-b) \end{pmatrix}\right\}^2 = \frac{1}{2}Cos^2(a-b)$$

$$P_{HH} = |\langle H|\langle H|\Pi(a,b)\,|BBO\rangle|^2 = \left\{(0 \quad 0 \quad 0 \quad 1)\frac{1}{\sqrt{2}}\begin{pmatrix} Cos(a-b) \\ Sin(a-b) \\ -Sin(a-b) \\ Cos(a-b) \end{pmatrix}\right\}^2 = \frac{1}{2}Cos^2(a-b)$$

$$P_{HV} = |\langle H|\langle V|\Pi(a,b)\,|BBO\rangle|^2 = \left\{(0 \quad 1 \quad 0 \quad 0)\frac{1}{\sqrt{2}}\begin{pmatrix} Cos(a-b) \\ Sin(a-b) \\ -Sin(a-b) \\ Cos(a-b) \end{pmatrix}\right\}^2 = \frac{1}{2}Sin^2(a-b)$$

$$P_{VH} = |\langle V|\langle H|\Pi(a,b)\,|BBO\rangle|^2 = \left\{(0 \quad 0 \quad 1 \quad 0)\frac{1}{\sqrt{2}}\begin{pmatrix} Cos(a-b) \\ Sin(a-b) \\ -Sin(a-b) \\ Cos(a-b) \end{pmatrix}\right\}^2 = \frac{1}{2}Sin^2(a-b)$$

(15)

And putting (15) into (8) yields

$$\begin{aligned} E(a,b) &= \frac{1}{2}Cos^2(a-b) + \frac{1}{2}Cos^2(a-b) - \frac{1}{2}Sin^2(a-b) - \frac{1}{2}Sin^2(a-b) \\ &= Cos^2(a-b) - Sin^2(a-b) \\ &= Cos2(a-b) \end{aligned}$$

(16)

Then finally we can simplify (4) as

$$S = \left|Cos2(a-b) - Cos2\left(a-\acute{b}\right)\right| + \left|Cos2(\acute{a}-b) + Cos2\left(\acute{a}-\acute{b}\right)\right|$$

(17)

This function will give the maximum value 2.82 at the following set of angles $a = 135°, \acute{a} = 0°, b = 157.5°, \acute{b} = 22.5°$.

Apart from this another interesting prediction is the first equation in (15), namely

$$P_{VV} = \frac{1}{2} Cos^2(a-b)$$

(18)

This result shows that the coincidence rate of entangled photons state is cosine squared dependent of the difference of the angles of two polarizers and the curve will have 100% visibility. The visibility is given by

$$Vis = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$$

(19)

where I_{max} and I_{min} are the maximum and minimum coincidence achieved by varying the angle of polarizers respectively.

Plug these in the theoretical maximum and minimum values from (18) we get the prediction

$$Vis = \frac{1-0}{1+0} \times 100\% = 100\%$$

(20)

Furthermore if we are interested in the count rates from single detector regardless of polarization of the other photon, we see that

$$\begin{aligned} P_{SINGLE} &= P_{VV} + P_{VH} = P_{HH} + P_{HV} \\ &= \frac{1}{2} Cos^2(a-b) + \frac{1}{2} Sin^2(a-b) \\ &= \frac{1}{2}\left(Cos^2(a-b) + Sin^2(a-b)\right) \\ &= \frac{1}{2} \end{aligned}$$

(21)

That is the single count rates from detectors are constant.

By verifying the violation of Bell's Inequalities we can prove the presence of entangled photons in our system and also these two predictions, cosine squared dependence and 100% visibility can be observed easily.

2. Set up, Experiments, and Results

2.1) Creating Entangled Photons [5]

Two BBO crystals type 1 are used as an entangled photons light source. We pump them with Ar-ion laser of wavelength 363.8 nm with definite polarization at 100 mW to produce down-converted photons of wave length 727.6 nm via parametric down-conversion process, SPDC. SPDC is a process which, in short, the crystal absorbs one photon to the excited state. With a small probability the crystal then spontaneously emits the photons by two successive jumping down to the intermediate level and then to the ground level, Fig 1. This process will create two lower energy photons. By cutting the crystal at an appropriate angle we can generate two photons of equal wavelength. The process creates two photons of the polarization perpendicular to the pumping photon, namely pumping with horizontal polarized photon will get two vertically polarized photons, and vice versa. Simplified mathematical expression of this process is

$$|H\rangle \rightarrow |V\rangle|V\rangle$$

Or

$$|V\rangle \rightarrow |H\rangle|H\rangle$$

(22)

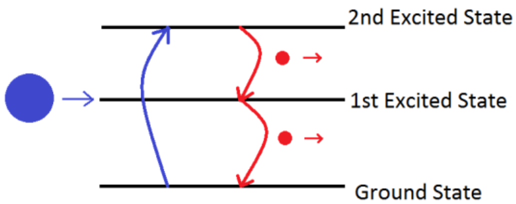

Figure 1 : Simplified energy diagram and SPDC process

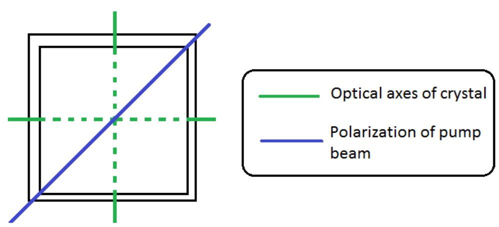

Figure 2 : Cross-section of alignment of two crystals and pump beam

To create entangled photons we need to put the two crystals so that their optical axes are perpendicular to each other and align the pump beam so that its polarization makes an angle of 45 degrees to the optical axes, Fig 2. This way the pumping photons have 50 percents chance to be converted by the first crystal and 50 percents by the second. This is not completely fulfilled the requirement for entangled photons yet. Since the photons produced by the second crystal will have phase shift with respect to the phase of the photons converted by the first crystal, namely

$$|45\rangle = \frac{1}{\sqrt{2}}\left(|V\rangle|V\rangle + e^{i\varphi}|H\rangle|H\rangle\right)$$

(23)

To compensate this factor we put the waveplate in front of the crystal to delay the phase of one component of two polarizations so that the down-converted photons will have zero phase shifts. With all of these procedures, we will obtain the entangled photons as we want. The entangled photons will emerge out off the crystals making light a light cone due to momentum conservation, Fig 3a

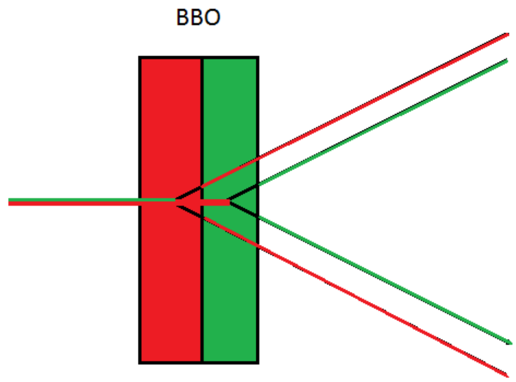

Figure 3 : Down-converted light cones. Red ray shows vertically polarized and green ray shows horizontally polarized. The color is not reflected to the true wavelength of photons

Apart from the desired entangled photon, we also have noise from the crystals which consists of two photons from, improperly, down-conversion with different wavelength mixing in the light in the cones. This noise can be removed by inserting two interference filters of band pass 727.6 nm with small bandwidth in front the detector. We also put, for safety, a beam stop to block the unconverted photons which also present after the crystals.

2.2)<u>Detectors</u>

We use two different photon detection systems in the experiment.

2.2.1) APDs and coincidence module

APDs are connected to the coincident count module to measure coincidence. The module is a computer board card connected directly to the computer. The signals from APDs are received by the module and number of photons detected in the experiment and coincidence among these photons will be reported. The time- window of the coincidence of this module is 26 ns, this time relates to the maximum separation in time of two photons detected by detectors that to be regarded as coincidence. This detecting system allows us to set the acquisition time and provides us number of photons found by each detector (single count) and coincidence count. This coincidence counting module will also give us the accidental coincidence count which can be estimated by the formula

$$AccidentalCoincidence = \frac{SingleCount\ A \times SingleCount\ B \times TimeWindow(26\ ns)}{AcquisitionTime} \quad (24)$$

2.2.2) CCD camera

This module consists of numerous CCD cells on the detection region of the detection. Each of them has its own amplifier and works independently. The detector provides us the number of photons found by each cell in the adjustable period of time. We can also construct the image of incident image using the specific program.

2.3)Experiment

We first verify the cosine squared dependence of the coincidence, see equation (18). APDs are used as a detection system in this part. On the other side of the optical table from BBO crystals, two APDs are placed to detect a photon from BBO's. By inserting two polarizers in front of each detector, we can measure coincidence rate of various combination of two polarization of each photons pair. Also the filters are inserted in front of them to reduce noise from improper down-conversion and background, Fig 4. For simplicity let called the detector on the right hand side as detector A and the left hand side detector as detector B. Also let call the polarizer in front detector A as polarizer A, and polarizer B for the other.

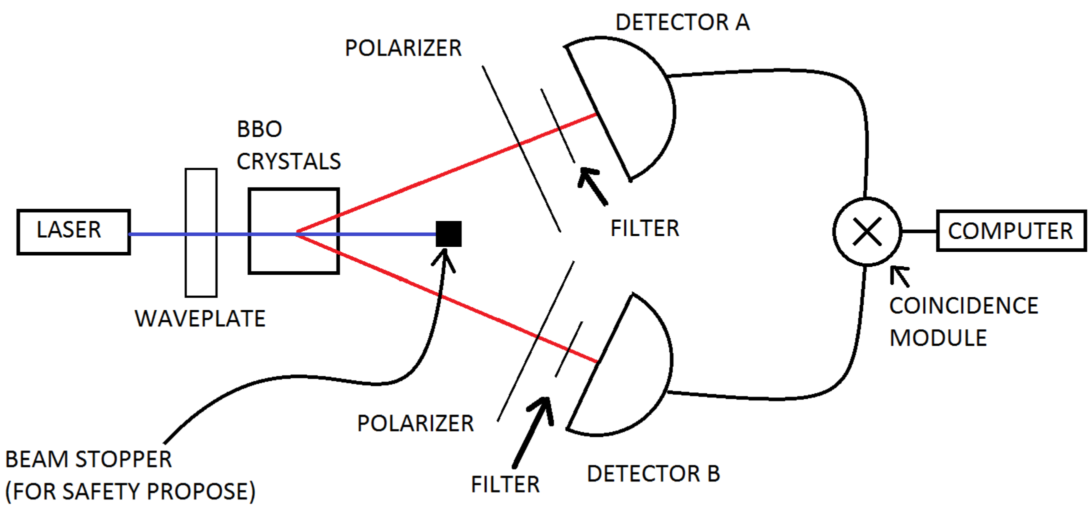

Figure 4 : Demonstrate all main parts of the set up.

First we set the polarizer A to be 45 degrees and the polarizer B to be 0 degrees. Before turning the laser on, measure background noise and dark count of each detector. We found zero background coincidence. Set acquisition time at 5 seconds. Record the following data 1) Single count from detector A. 2) Count rate of detector B. 3) Coincidence. Repeat it three times and find their average. Then change the polarizer B to 10 degrees and measure those quantities again. Repeat this step for every 10 degrees increment of the angle of polarizer B until we rotate it by 360 degrees. Then rotate the polarizer A to 135 degrees and repeat the previous step again. We then subtract accidental coincidence from measured coincidence using equation (23). By plotting the net coincidence vs. angle of polarizer B, cosine squared dependences are clearly seen, Fig 5. Also another interesting result is the count rate from each detector which are shown in Fig 6. Notice that the count rates are fairly constant with visibility about 17% which agrees with (21). And finally we calculate the visibility of coincidence each set to be at 98.2% for polarizer A at 45 degrees and 99.5% for polarizer A at 135 degrees which are close to 100% as expected.

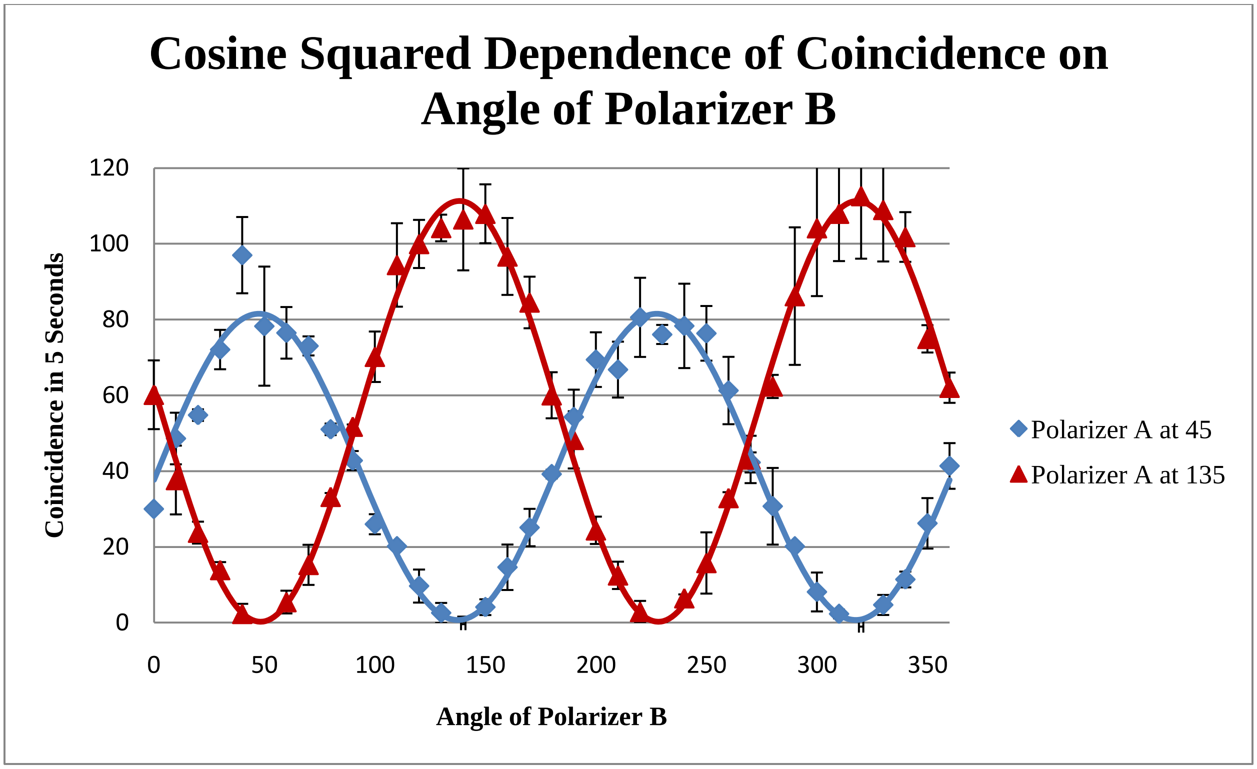

Figure 5 : Plots of coincidence as a function of angle of polarizer B

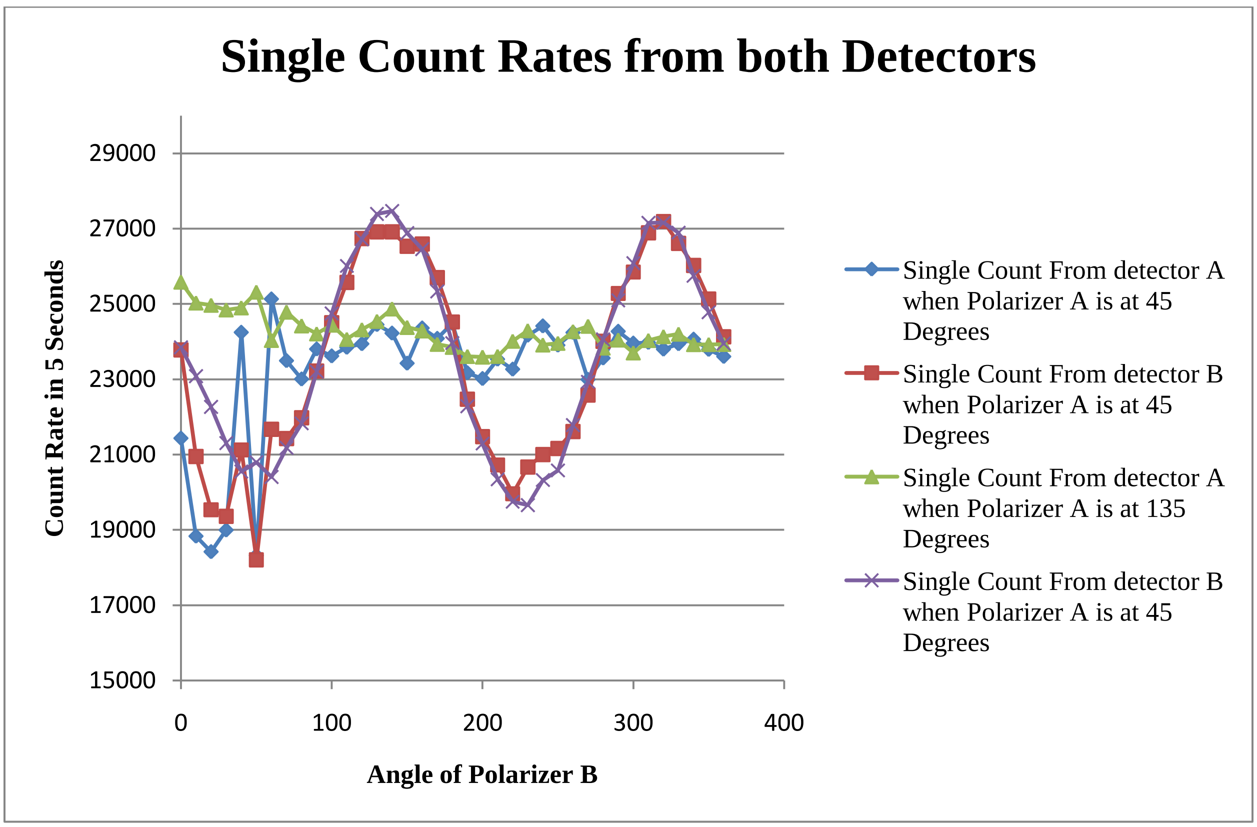

Figure 6: Plots of Count Rates from each Detectors as a function of angle of polarizer B

We then check the violation of Bell's Inequalities. We measure single counts and coincidence for 20 second for three times when the polarizer A is at 0 degrees and polarizer B is at 22.5 degrees. We then repeat this procedure for polarizer B at 67.5, 112.5, and 157.5 degrees. We then change polarizer A to 45, 90, and 135 degrees, each time measure coincidence when polarizer B is at one of the four angles. We should end up with 16 pairs of angles. Subtract the accidental coincidence, equation (23), and find the average and standard deviation of coincidence from each pair of angles. The result is shown in Table 1.

Table 1 : Coincidence in Verifying Bell's Inequalities

AngleOf	AngleOf	1st Trial					2nd Trial					3rd Trial					Average		SD	
Polari-zationA	Polari-zationB	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Coinci-dence	Net	Coinci-dence	Net
0	22.5	88255	97272	253	11	242	88255	97272	256	11	245	88255	97272	222	11	211	244	233	19	19
0	67.5	88670	98447	91	11	80	88670	98447	103	11	92	88670	98447	94	11	83	96	85	6	6
0	112.5	93516	97795	165	12	153	93516	97795	151	12	139	93516	97795	146	48	98	154	130	10	28
0	157.5	84740	89381	296	10	286	84740	89381	242	10	232	84740	89381	267	39	228	268	249	27	33
45	22.5	81171	74592	279	8	271	81171	74592	286	8	278	81171	74592	259	31	228	275	259	14	27
45	67.5	77660	73401	234	7	227	77660	73401	221	7	214	77660	73401	238	30	208	231	216	9	9
45	112.5	88844	77887	51	9	42	88844	77887	39	9	30	88844	77887	83	36	47	58	40	23	9
45	157.5	88053	76084	71	9	62	88053	76084	66	9	57	88053	76084	35	35	0	57	40	20	35
90	22.5	90654	96811	140	11	129	90654	96811	122	11	111	90654	96811	138	46	92	133	111	10	18
90	67.5	90148	93100	267	11	256	90148	93100	271	11	260	90148	93100	239	44	195	259	237	17	36
90	112.5	90094	89758	271	11	260	90094	89758	243	11	232	90094	89758	251	42	209	255	234	14	26
90	157.5	92459	92044	117	11	106	92459	92044	142	11	131	92459	92044	150	44	106	136	114	17	14
135	22.5	96572	117723	48	15	33	96572	117723	54	15	39	96572	117723	56	59	-3	53	23	4	23
135	67.5	97563	118603	161	15	146	97563	118603	157	15	142	97563	118603	145	60	85	154	124	8	34
135	112.5	92178	108582	408	13	395	92178	108582	378	13	365	92178	108582	391	52	339	392	366	15	28
135	157.5	85934	100883	313	11	302	85934	100883	351	11	340	85934	100883	335	45	290	333	310	19	26

From these data we calculate the quantities quantity S using equation (8) and (9). We obtain $S = 1.98 \pm 0.17$. This result does not completely violate the inequalities, nevertheless the uncertainty of the quantity on plus side is greater that two. This suggests that some devices in the set up are not aligned properly. We narrow all the possibilities down to three choices. 1.) The BBO crystals 2.) The position of the detector. and 3.) The waveplate used in compensating the phase shift

We first investigate the BBO crystals and find out that they are slightly misaligned. After re-aligning the crystals, we observed the down-converted photons light cones, Fig 3, by placing CCD camera behind the crystals. Lens is used to scale the image so that it is small enough for the detection region of the CCD camera. We can capture the image of down-converted light cone as expected, Fig 7.

After that we check the positions of two detectors so that both of them detecting the photons from the opposite sides of the down-converted light cone. This part is done with help from Dr. Lukishova.

Next experiment is to optimize the phase shift between two two-photon states, equation (22), to achieve a better entangled photons state which will yield better results. Optimizing is done by adjusting the angles of waveplate both vertical axis and horizontal axis. We search for the case when the measured coincidence is independent of the angle of polarizers, equation (7). We carry out this experiment by moving back to use APDs detectors and two polarizers. We measure coincidence from the following pairs of angles of polarizers, 0-0, 45-45, 90-90, and 135-135 with acquisition time of 5 second. We first optimize vertical angle in the range of 50 degrees with 2 degrees increment and fix horizontal angle at 359 degrees. We obtain the optimized angle at 36 degrees, fig 7. We then move to optimize the horizontal angle by fixing vertical angle at 36 degrees and we get the optimized angle at 359 degrees, fig 8. For more precise result, we start the optimizing again with narrower range and finer increment, 6 degrees range for vertical and 10 degrees for horizontal with increment of 1 degree. The optimized angles we get are 36.5 degrees for vertical angle and 359.5 degrees for horizontal, fig 9 and fig 10.

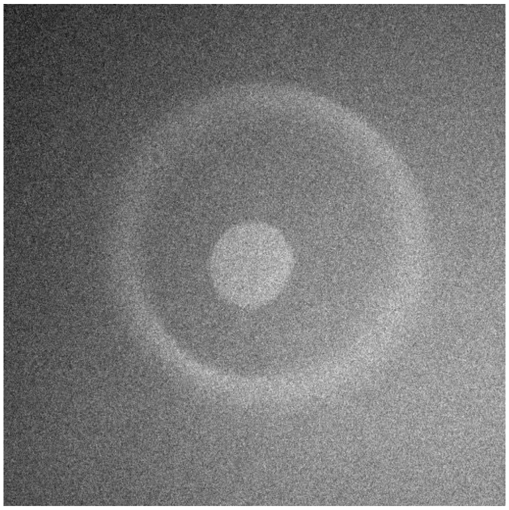

Figure 7 : The image of down-converted light is

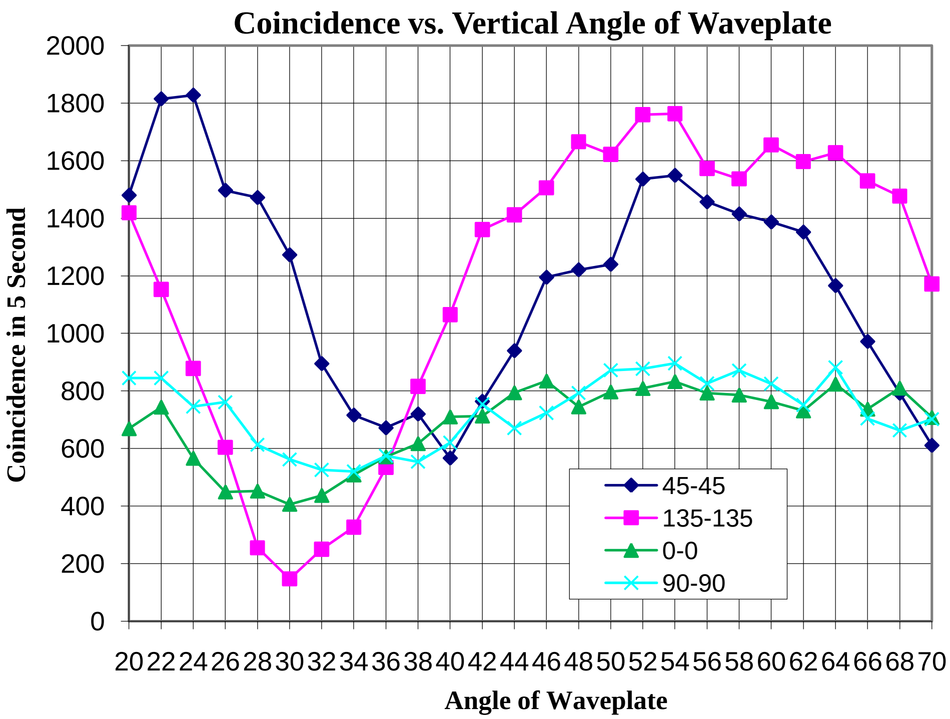

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Figure 8 : Optimizing Vertical Angle

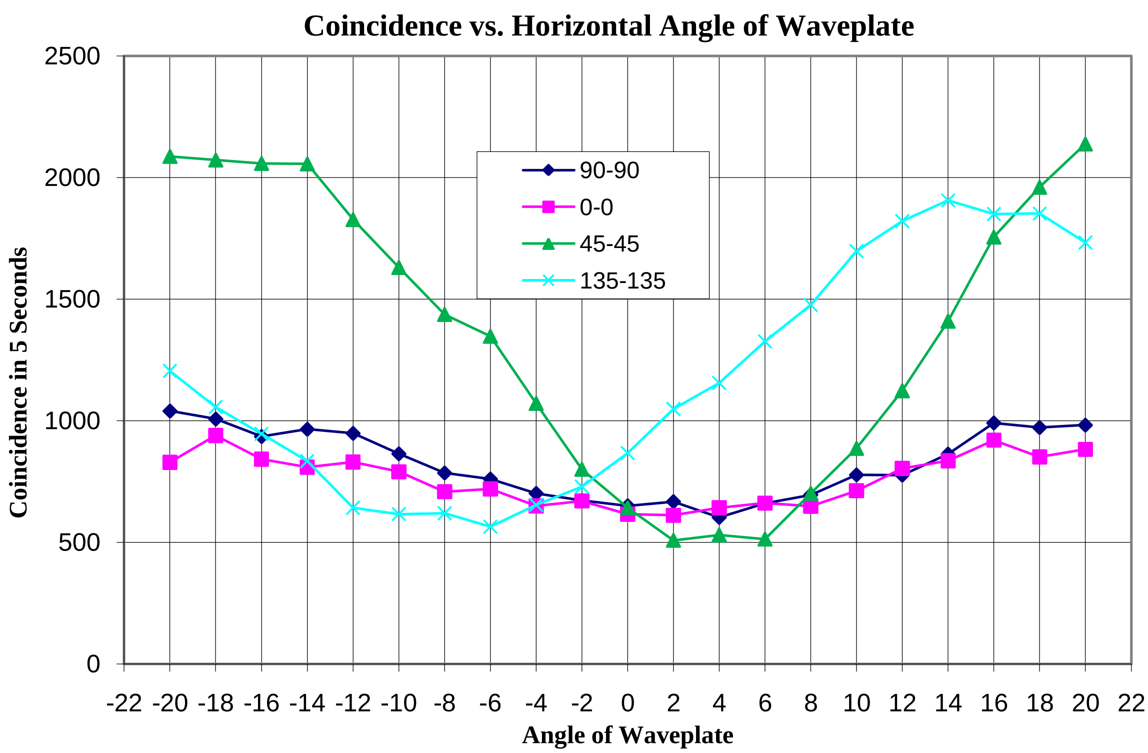

Figure 9 : Optimizing Horizontal Angle

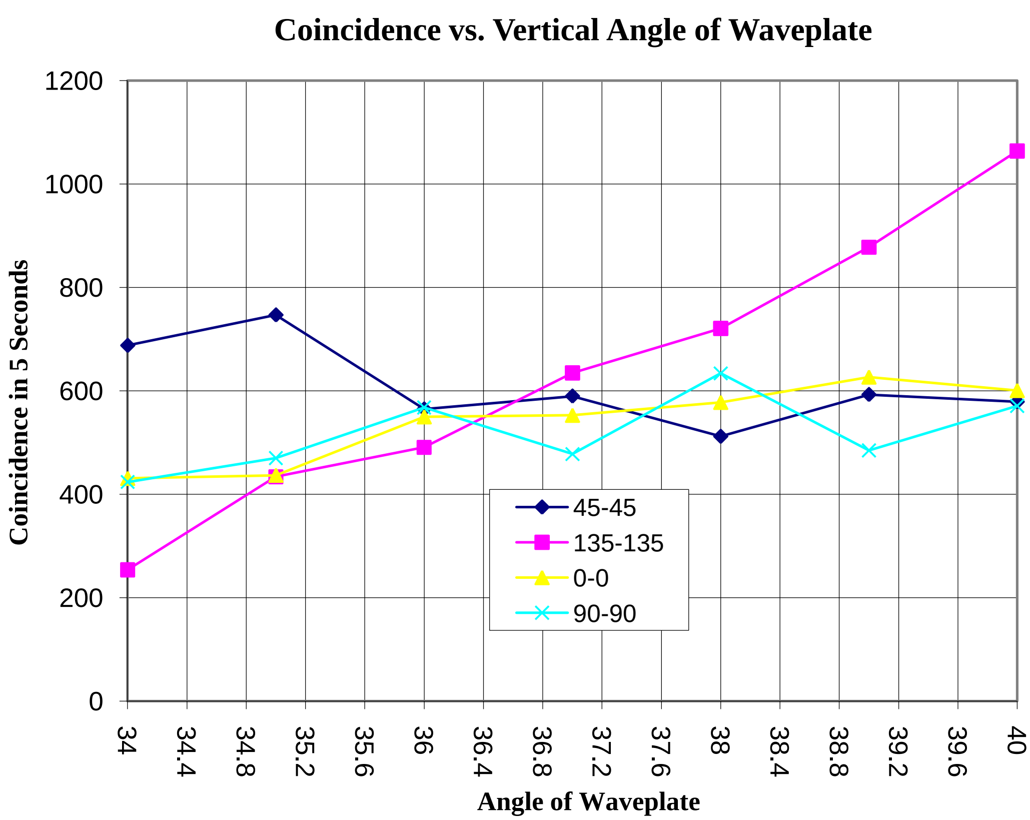

Figure 10: Finer Optimizing Vertical Angle

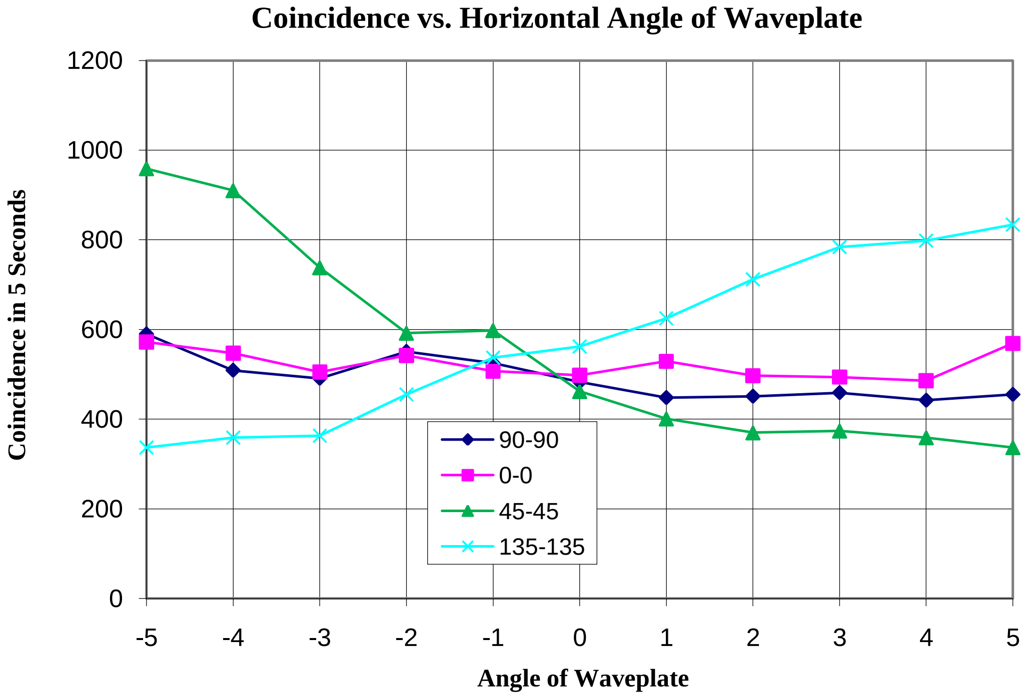

Figure 11 : Finer Optimizing Horizontal Angle

Next we verify the visibility approximately by measuring coincidence for 5 seconds when both polarizers are at the same angles, which is theoretically the maximum, equation (18). Then rotate one of them by 90 degrees and measure the coincidence which is now the minimum theoretically, again by equation (18). We pick the following angles 0, 45, 16, and 100. The results for visibility are 81%, 96%, 88% and 81%, respectively, as shown in Table 2. Obviously, they are all close to 100% in an acceptable range.

Table 2: Approximate Visibility After Optimizing Waveplate

Angles (Degrees)	Maximum Coincidence (Count)	Minimum Coincidence (Count)	Visibility (%)
0,90	555.00	59	81
45,135	611	13	96
16,106	530	33	88
100,190	507.00	53	81

Finally we check the violation of Bell's inequalities once again after optimizing all of the devices by carrying out the measurement coincidence for 16 pairs of angles of polarizers. The data obtained this time is shown in Table 3.

Table 3 : Coincidence in Verifying Bell's Inequalities

AngleOf	AngleOf	1st Trial					2nd Trial					3rd Trial					Average		SD	
Polari-zationA	Polari-zationB	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Single CountA	Single CountB	Coinci-dence	Acci-dental	Net	Coinci-dence	Net	Coinci-dence	Net
0	22.5	41813	32753	384	7	377	41362	32378	307	7	300	42054	32658	363	7	356	351	344	40	40
0	67.5	41717	33007	100	7	93	41398	32442	105	7	98	41229	32519	102	7	95	102	95	3	3
0	112.5	41597	33288	55	7	48	41146	33160	79	7	72	41895	33462	87	7	80	74	66	17	17
0	157.5	41708	33101	312	7	305	41869	33566	310	7	303	41445	32906	306	7	299	309	302	3	3
45	22.5	46468	36537	403	9	394	46932	36412	373	9	364	46639	36135	369	9	360	382	373	19	19
45	67.5	41238	32781	309	7	302	40764	32016	348	7	341	40837	32194	327	7	320	328	321	20	20
45	112.5	41459	33505	71	7	64	41175	32917	86	7	79	41442	33039	83	7	76	80	73	8	8
45	157.5	41946	33293	42	7	35	41788	33252	44	7	37	41764	33142	51	7	44	46	38	5	5
90	22.5	46391	37309	99	9	90	45764	36720	87	9	78	46416	37333	82	9	73	89	80	9	9
90	67.5	46115	37632	390	9	381	47504	38383	391	9	382	46905	38308	373	9	364	385	375	10	10
90	112.5	47849	38984	423	10	413	47731	39452	408	10	398	47127	39266	391	10	381	407	398	16	16
90	157.5	46668	38168	134	9	125	47658	38210	124	9	115	47053	38215	122	9	113	127	117	6	7
135	22.5	45759	37576	111	9	102	47443	38686	103	10	93	47465	38623	103	10	93	106	96	5	5
135	67.5	48317	39571	51	10	41	48288	39508	50	10	40	47966	39372	56	10	46	52	42	3	3
135	112.5	47410	39216	372	10	362	47706	39494	417	10	407	47234	38993	383	10	373	391	381	23	23
135	157.5	46982	38442	453	9	444	47506	38012	476	9	467	46853	38204	465	9	456	465	455	12	12

Then we calculate quantity S again using equation (8) and (9). We now obtain $S = 2.64 \pm 0.08$ which obviously violates the inequalities. This result implies that we really achieve the entangled photons as we expect.

3. Conclusion

We obtain $S = 2.64 \pm 0.08$ which is greater than 2 and violates Bell's Inequalities. This result strongly confirms that we have produced entangled photons by using SPDC process in BBO type 1 crystals. Furthermore the cosine squared dependence and visibility about 100% are also confirmed.

4. References

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